GOVERNMENT OF ANDHRA PRADESH

COMMISSIONERATE OF COLLEGIATE EDUCATION

LINEAR ALGEBRA- INNEPRODUCT SPACE

ORTHOGONALITY

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Definition:

Let α, β be vectors in an inner product space V(f).Then α is said to be orthogonal to β if $(\alpha, \beta) = 0$

Ex: If $\alpha =$ (1 ,1, 1) and $\beta =$ (1,0,-1)

$$(\alpha, \beta) = 1 + 0 - 1 = 0$$

Note (a).The relation of orthogonal in an inner product space is symmetric.

i.e $(\alpha, \beta) = 0 \Longrightarrow (\beta, \alpha) = 0$

Therefore $\beta\ is\ \ orthogonal\ to\ \alpha$

Note (b).1). If $\alpha\ is\ orthogonal\ to \beta, then$ any scalar multiple of $\alpha\ is\ \ also\ \ othogonal\ to \beta$.

i.e (k$\alpha, \beta) = k(\alpha, \beta) = k.0 = 0$

Therefore k$\alpha\ is\ orthogonal\ to\ \beta$

Therefore k$\alpha\ is\ orthogonal\ to\ \beta$

2).The zero vector is orthogonal to every vector (0,$\alpha) = 0$

Theorem: The vectors α and β in a real inner product space are orthogonal if and only if $\| \alpha+\beta \|^2 = \| \alpha \|^2 + \| \beta \|^2$

Proof: Let α and β are vectors in inner product space V(F).

$$\text{Now} \quad \| \alpha+\beta \|^2 = \| \alpha \|^2 + \| \beta \|^2$$

$$\Leftrightarrow (\alpha+\beta, \alpha+\beta) = (\alpha, \alpha) + (\beta, \beta)$$

$$\Leftrightarrow (\alpha.\alpha) + (\alpha, \beta) + (\beta, \alpha) + (\beta, \beta) = (\alpha, \alpha) + (\beta, \beta)$$

$\Leftrightarrow (\alpha, \beta) + (\beta, \alpha) = 0$

$\Leftrightarrow (\alpha, \beta) + (\alpha, \beta) = 0$ $\because$ Real I. P.S

$\Leftrightarrow 2(\alpha, \beta) = 0$

$\Leftrightarrow (\alpha, \beta) = 0$

$\Leftrightarrow \alpha, \beta$ are orthogonal

Hence the proof

Definition: **Orthogonal set:**

Let V(F) be an inner product space. A nonempty sub set S of V is said to be an orthogonal set if any two distinct vectors in S are orthogonal. (OR)

Let S = { $\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_n$ } is orthogonal subset of an inner product space if

$(\alpha_i, \alpha_j) = 0$ if $i \neq j$ and

$(\alpha_i, \alpha_j) = 1$ if $i = j$ in V(F)

Theorem: Every orthogonal set of non zero vectors in an inner product space V(F) is linearly independent.

Proof: Let S be a an orthogonal set of non zero vectors in an inner product space V(F).

Let $S_1 = \{ \alpha_1, \alpha_2, \alpha_3, \ldots \alpha_n \}$ be finite subset of S , contain 'n' non zero vectors

$$\Rightarrow \quad a_1\alpha_1 + a_2\alpha_2 + \ldots\ldots\ldots + a_n\alpha_n = 0$$

Let $0 = (0, \alpha_k)$

$\Rightarrow (a_1\alpha_1, + a_2\alpha_2 + , \ldots\ldots + a_n\alpha_n, \alpha_k) = (\bar{0}, \alpha_k)$

$\Rightarrow a_1(\alpha_1, \alpha_k) + a_2(\alpha_2, \alpha_k) + \ldots + a_k(\alpha_k, \alpha_k) +$

$+ \ldots\ a_n(\alpha_n, \alpha_k) = \bar{0}$

$\Rightarrow a_1(0) + a_2(0) + \ldots + a_k(\alpha_k, \alpha_k) + \ldots\ldots + a_n(0) = 0$

$\Rightarrow a_k(\alpha_k, \alpha_k) = \bar{0}$

$\Rightarrow a_k \parallel \alpha_k \parallel^2 = \bar{0}$

$\Rightarrow a_k = \bar{0}, \quad (\because \forall 1 \le k \le n)$

$\therefore$ s_1 is linearly independent

$\therefore$ Every finite subset of S is linearly independent

$\therefore$ Every orthogonal set of nonzero vectors in an inner product is space V(F)is linearly independent.

THANK YOU